DAY SIX

Work, Energy and Power

Learning & Revision for the Day

- Work
- Conservative and Non-conservative Force
- Energy
- Work-Energy Theorem
- Law of Conservation of Energy
- Power
- Collision

Work

Work is said to be done, when a body a displaced through some distance in the direction of applied force. The SI unit work is joule (J) and in CGS it is erg.

$$1 \text{ joule (J)} = 10^7 \text{ erg}$$

The work done by the force **F** in displacing the body through a distance **s** is

$$W = (F\cos\theta)s = Fs\cos\theta = \mathbf{F}\cdot\mathbf{s}$$

where, $F\cos\theta$ is the component of the force, acting along the direction of the displacement produced. SI unit of work is joule (J).

$$1\text{ J} = 1\text{ N-m}$$

Work is a scalar quantity. Work can be of three types

(i) Positive work (ii) Negative work and (iii) Zero work.

- **Positive work** If value of the angle θ between the directions of **F** and **s** is either zero or an acute angle.
- **Negative work** If value of angle θ between the directions of **F** and **s** is either 180° or an obtuse angle.
- As work done $W = \mathbf{F}\cdot\mathbf{s} = Fs\cos\theta$, hence **work done can be zero,** if

(i) No force is being applied on the body, i.e. $F = 0$.

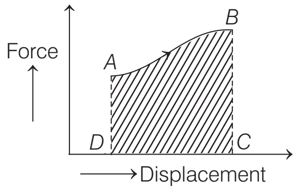
(ii) Although the force is being applied on a body but it is unable to cause any displacement in the body, i.e. $F \neq 0$ but $s = 0$.

(iii) Both F and s are finite but the angle θ between the directions of force and displacement is 90°. In such a case

$$W = \mathbf{F}\cdot\mathbf{s} = Fs\cos\theta = Fs\cos 90° = 0$$

Work Done by Variable Force

Work done by a variable force is given by $W = \int \mathbf{F} \cdot d\mathbf{s}$



It is equal to the area under the force-displacement graph, along with proper sign.

Work done = Area of *ABCDA*

Conservative and Non-conservative Force

A force is said to be **conservative** if work done by or against the force in moving a body depends only on the initial and final positions of the body and not on the nature of path followed between the initial and the final position. Gravitational force, force of gravity, electrostatic force are some examples of conservative forces (fields).

A force is said to be **non-conservative** if work done by or against the force in moving a body from one positions to another, depends on the path followed between these two positions. Force of friction and viscous force are the examples of non-conservative forces.

Energy

Energy is defined as the capacity or ability of a body to do work. Energy is scalar and its units and dimensions are the same as that of work. Thus, SI unit of energy is J.

Some other commonly used units of energy are

$$1 \text{ erg} = 10^{-7} \text{ J},$$

$$1 \text{ cal} = 4.186 \text{ J} \cong 4.2 \text{ J},$$

$$1 \text{ kcal} = 4186 \text{ J},$$

$$1 \text{ kWh} = 3.6 \times 10^6 \text{ J},$$

and 1 electron volt $(1 \text{ eV}) = 1.60 \times 10^{-19}$ J

Kinetic Energy

- Kinetic energy is the capacity of a body to do work by virtue of its motion. A body of mass m, moving with a velocity v, has a kinetic energy, $K = \frac{1}{2} mv^2$.
- Kinetic energy of a body is always positive irrespective of the sign of velocity v. Negative kinetic energy is impossible. Kinetic energy is correlated with momentum as,

$$K = \frac{p^2}{2m} \quad \text{or} \quad p = \sqrt{2mK}$$

- Kinetic energy for a system of particle will be

$$K = \frac{1}{2} \sum_i m_i v_i^2$$

- Relation between kinetic energy and force is

$$\frac{\text{KE}}{\text{Force}} = \frac{v \times t}{2}$$

where, v is velocity and t is time.

Potential Energy

Potential energy is the energy stored in a body or a system by virtue of its position in a field of force or due to its configuration. Potential energy is also called **mutual energy** or **energy of the configuration**.

Value of the potential energy in a given position can be defined only by assigning some arbitrary value to the reference point. Generally, reference point is taken at infinity and potential energy at infinity is taken as zero. In that case,

$$U = -W = -\int_{\infty}^{r} \mathbf{F} \cdot d\mathbf{r}$$

Potential energy is a scalar quantity. It may be positive as well as negative.

Different types of potential energy are given below.

Gravitational Potential Energy

It is the energy associated with the state of separation between two bodies which interact *via* the gravitational force.

- The gravitational potential energy of two particles of masses m_1 and m_2 separated by a distance r is

$$U = \frac{-Gm_1m_2}{r}.$$

- If a body of mass m is raised to a height h from the surface of the earth, the change in potential energy of the system (earth+body) comes out to be

$$\Delta U = \frac{mgh}{\left(1 + \frac{h}{R}\right)}$$

or $\quad \Delta U \approx mgh$ if $h << R$

Thus, the potential energy of a body at height h, i.e. mgh is really the change in potential energy of the system for $h << R$.

- For the gravitational potential energy, the zero of the potential energy is chosen on the ground.

Electric Potential Energy

The electric potential energy of two point charges q_1 and q_2 separated by a distance r in vacuum is given by

$$U = \frac{1}{4\pi\varepsilon_0} \frac{q_1q_2}{r}$$

where, $\frac{1}{4\pi\varepsilon_0} = 9.1 \times 10^9 \frac{\text{N-m}^2}{\text{C}^2} = \text{constant}$

Potential Energy of a Spring

Whenever an elastic body (say a spring) is either stretched or compressed, work is being done against the elastic spring force. The work done is $W = \frac{1}{2}kx^2$,

where, k is spring constant and x is the displacement.

And elastic potential energy, $U = \frac{1}{2}kx^2$

If spring is stretched from initial position x_1 to final position x_2, then

work done = Increment in elastic potential energy

$$= \frac{1}{2}k(x_2^2 - x_1^2).$$

Work-Energy Theorem

Accordingly, work done by all the forces (conservative or non-conservative, external or internal) acting on a particle or an object is equal to the change in its kinetic energy of the particle. Thus, we can write

$$W = \Delta K = K_f - K_i$$

We can also write, $K_f = K_i + W$

Which says that $\begin{pmatrix}\text{Kinetic energy after}\\ \text{the net work is done}\end{pmatrix}$

$$= \begin{pmatrix}\text{Kinetic energy before}\\ \text{the net work done}\end{pmatrix} + \begin{pmatrix}\text{The net}\\ \text{work done}\end{pmatrix}$$

Law of Conservation of Energy

The mechanical energy E of a system is the sum of its kinetic energy K and its potential energy U.

$$E = K + U$$

When the forces acting on the system are conservative in nature, the mechanical energy of the system remains constant,

$$K + U = \text{constant} \Rightarrow \Delta K + \Delta U = 0$$

There are physical situations, where one or more non-conservative force act on the system but net work done by them is zero, then too the mechanical energy of the system remains constant.

If $\qquad \Sigma W_{\text{net}} = 0$

Mechanical energy, E = constant.

Power

- It is a quantity that measures the rate at which work is done or energy is transformed.

  Average power $(P)_{\text{av}} = \frac{W}{t}$

- The shorter is the time taken by a person or a machine in performing a particular task, the larger is the power of that person or machine.
- Power is a scalar quantity and its SI unit is watt, where, 1W = 1 J/s
- Instantaneous power,

$$\mathbf{P}_{\text{inst}} = \frac{dW}{dt} = \frac{\mathbf{F} \cdot d\mathbf{s}}{dt} = \mathbf{F} \cdot \mathbf{v}$$

- Some other commonly used units of power are

  $1\text{ kW} = 10^3\text{ W}$,

  $1\text{ MW} = 10^6\text{ W}$,

  $1\text{ HP} = 746\text{ W}$

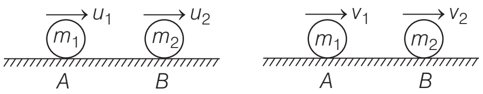
Collision

The physical interaction of two or more bodies in which each equal and opposite forces act upon each other causing the exchange of energy and momentum is called collision. Collisions are classified as

(i) elastic collisions and (ii) inelastic collisions.

Elastic Collision in One Dimension

In a perfectly elastic collision, total energy and total linear momentum of colliding particles remains conserved. Moreover, the forces involved in interaction are conservative in nature and the total kinetic energy before and after the collision, remains unchanged.



Before collision After collision

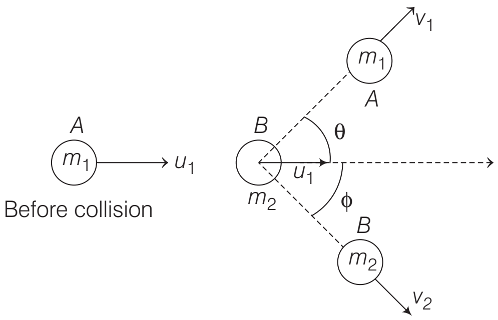
In above figure, two bodies A and B of masses m_1 and m_2 and having initial velocities u_1 and u_2 in one dimension, collide elastically and after collision move with velocities v_1 and v_2, then we find that

- Relative velocity of approach = Relative velocity of separation, i.e. $u_1 - u_2 = v_2 - v_1$
- $v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1 + \left(\frac{2m_2}{m_1 + m_2}\right)u_2$

  and $v_2 = \left(\frac{2m_1}{m_1 + m_2}\right)u_1 + \left(\frac{m_2 - m_1}{m_1 + m_2}\right)u_2$

Elastic Collision in Two Dimensions

- In this type of collision, the two particles or objects moving along different directions collide with each other.



Before collision After collision

- As linear momentum is conserved.

  ∴ Along the x-axis

$$m_1u_1 + m_2u_2 = m_1v_1\cos\theta + m_2v_2\cos\phi \quad \text{...(i)}$$

and along the y-axis

$$0 = m_1v_1\sin\theta - m_2v_2\sin\phi \quad \text{...(ii)}$$

- As the total kinetic energy remains unchanged.

  Hence, $$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 \quad \text{...(iii)}$$

  We can solve these equations provided that either the value of θ and ϕ is known to us.

Inelastic Collision in One Dimension

In an inelastic collision, the total linear momentum as well as total energy remain conserved but total kinetic energy after collision is not equal to kinetic energy before collision.

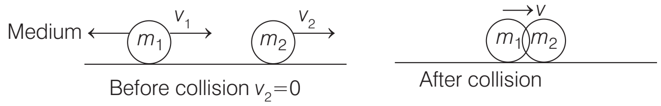
For inelastic collision,

Common speed, $v = \dfrac{m_1v_1}{m_1 + m_2}$

and loss of kinetic energy $\Delta K = \dfrac{m_1m_2(v_1 - v_2)^2}{2(m_1 + m_2)}$

Here $v_2 = 0$

∴ $$\Delta K = \frac{m_1m_2v_1^2}{2(m_1 + m_2)}$$



Coefficient of Restitution (e)

- It is defined as the ratio of relative velocity of separation to the relative velocity of approach.
- Coefficient of restitution, $e = \dfrac{v_2 - v_1}{u_1 - u_2}$
- For a perfectly elastic collision, $e = 1$.
- For a perfectly inelastic collision, $e = 0$.
- If $0 < e < 1$, the collision is said to be **partially elastic.**
- In a perfectly inelastic collision, $e = 0$ which means that $v_2 - v_1 = 0$ or $v_2 = v_1$.
- It can be shown that for an inelastic collision the final velocities of the colliding bodies are given by

$$v_1 = \left(\frac{m_1 - em_2}{m_1 + m_2}\right)u_1 + \frac{(1 + e)m_2}{(m_1 + m_2)}u_2$$

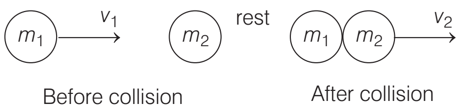
and $$v_2 = \frac{(1 + e)m_1}{(m_1 + m_2)}u_1 + \left(\frac{m_2 - em_1}{m_1 + m_2}\right)u_2$$

- If a particle of mass m, moving with velocity u, hits an identical stationary target inelastically, then final velocities of projectile and target are correlated as

  i.e. $m_1 = m_2 = m$ and $u_2 = 0$;

$$\frac{v_1}{v_2} = \frac{1 - e}{1 + e}$$

- In case of extreme inelastic collision (in which colliding objects stick together after collision)



- Final velocity $(v_2) = \left[\dfrac{m_1}{m_1 + m_2}\right]v_1$
- Ratio of kinetic energy before and after collision is

$$\frac{\text{KE}_1}{\text{KE}_2} = \left[\frac{m_1}{m_1 + m_2}\right]$$

- Fraction of kinetic energy lost in the collision is

$$\frac{\text{KE}_1 - \text{KE}_2}{\text{KE}_1} = \left[\frac{m_2}{m_1 + m_2}\right]$$

Rebounding of a Ball on Collision with the Floor

- Speed of the ball after the nth rebound

$$v_n = e^n v_0 = e^n\sqrt{2gh_0}$$

- Height covered by the ball after the nth rebound

$$h_n = e^{2n}h_0$$

- Total distance (vertical) covered by the ball before it stops bouncing

$$H = h_0 + 2h_1 + 2h_2 + 2h_3 + \ldots = h_0\left(\frac{1 + e^2}{1 - e^2}\right)$$

- Total time taken by the ball before it stops bouncing

$$T = t_0 + t_1 + t_2 + t_3 + \ldots$$

$$= \sqrt{\frac{2h_0}{g}} + 2\sqrt{\frac{2h_1}{g}} + 2\sqrt{\frac{2h_2}{g}} + \ldots$$

$$= \sqrt{\frac{2h_0}{g}}\left(\frac{1 + e}{1 - e}\right)$$

DAY PRACTICE SESSION 1

FOUNDATION QUESTIONS EXERCISE

1 A body of mass 2 kg, initially at rest, is acted upon simultaneously by two forces, one of 4N and other of 3N, acting at right angles to each other. The work done by the body after 20 s is

(a) 500 J (b) 1250 J (c) 2500 J (d) 5000 J

2 A body of mass 500 g is taken up an inclined plane of length 10 m and height 5 m and then released to slide down to the bottom. The coefficient of friction between the body and the plane is 0.1. What is the amount of work done in the round trip?

(a) 5 J (b) 15 J (c) $5\sqrt{3}$ J (d) $\frac{5}{\sqrt{3}}$ J

3 A block of mass 5 kg is initially at rest on a horizontal frictionless surface. A horizontal force $\mathbf{F} = (9 - x^2)\,\hat{\mathbf{i}}$ Newton acts on it, when the block is at $x = 0$. The maximum work done by the block between $x = 0$ and $x = 3$ m in joule is

(a) 18 J (b) 15 J (c) 20 J (d) 24 J

4 An object is displaced from point A (2m, 3m, 4m) to a point B (1m, 2m, 3m) under a constant force $\mathbf{F} = (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}})$N, then the work done by this force in this process is

(a) 9 J (b) –9 J (c) 18 J (d) –18 J

5 An open water tight railway wagon of mass 5×10^3 kg coasts with an initial velocity of 1.2 ms^{-1} on a railway track without friction. Rain falls vertically downwards on the wagon.

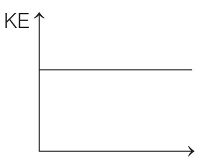
What change occurs in the kinetic energy of the wagon, after it has collected 10^3 kg of water?

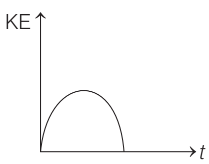
(a) 900 J (b) 300 J (c) 600 J (d) 1200 J

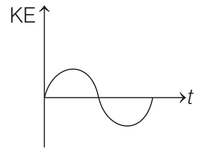
6 A particle moves in a straight line with retardation proportional to its displacement. The loss in kinetic energy of the particle, for any displacement x, is proportional to

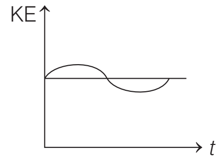
(a) x^2 (b) e^x (c) x (d) $\log_e x$

7 Which of the diagrams as shown in figure most closely shows the variation in kinetic energy of the earth as it moves once around the sun in its elliptical orbit?

(a)


(b)


(c)


(d)


8 A particle is moving in a circular path of radius a under the action of an attractive potential $U = -\frac{k}{2r^2}$. Its total energy is

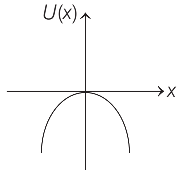
(a) $-\frac{k}{4a^2}$ (b) $\frac{k}{2a^2}$ (c) zero (d) $-\frac{3}{2}\frac{k}{a^2}$

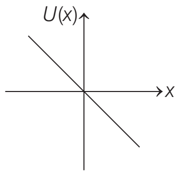
9 The potential energy of a 1 kg particle free to move along the x-axis is given by $V(x) = \left(\frac{x^4}{4} - \frac{x^2}{2}\right)$ J.

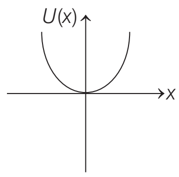
The total mechanical energy of the particle is 2 J. Then, the maximum speed (in ms^{-1}) is

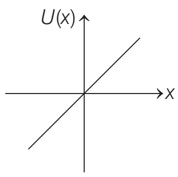
(a) $\frac{3}{\sqrt{2}}$ (b) $\sqrt{2}$ (c) $\frac{1}{\sqrt{2}}$ (d) 2

10 A particle is placed at the origin and a force $F = kx$ acts on it (where, k is a positive constant). If $U(0) = 0$, the graph of $U(x)$ *versus* x will be (where, U is the potential energy function)

(a)


(b)


(c)


(d)


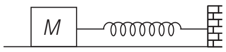
11 A bullet fired into a fixed target losses half of its velocity after penetrating distance of 3 cm. How much further it will penetrate before coming to rest, assuming that it faces constant resistance to its motion?

(a) 3.0 cm (b) 2.0 cm (c) 1.5 cm (d) 1.0 cm

12 A 1.5 kg block is initially at rest on a horizontal frictionless surface, when a horizontal force in the positive direction of x-axis, is applied to the block. The force is given by $\mathbf{F} = (4 - x^2)\,\hat{\mathbf{i}}$ Newton, where x is in metre and the initial position of the block is at $x = 0$. The maximum kinetic energy of the block between $x = 0$ and $x = 2.0$ m is

(a) 6.67 J (b) 5.33 J (c) 8.67 J (d) 2.44 J

13 A block of mass M moving on a frictionless horizontal surface, collides with a spring of spring constant k and compresses it by length L. The maximum momentum of the block, after collision is



(a) $L\sqrt{Mk}$ (b) $\frac{kL^2}{2M}$ (c) zero (d) $\frac{ML^2}{k}$

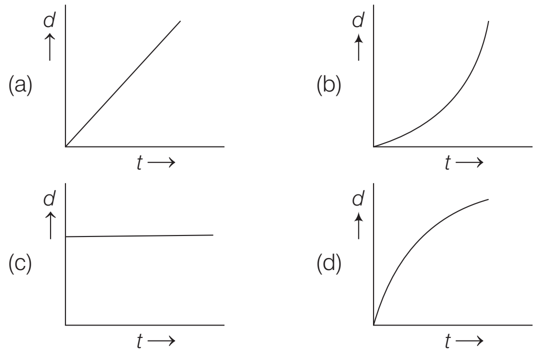
14 A cyclist rides up a hill with a constant velocity. Determine the power developed by the cyclist, if the length of the connecting rod of the pedal is $r = 25$ cm, the time of revolution of the rod is $t = 2$ s and the mean force exerted by his foot on the pedal is $F = 15$ kgf.

(a) 115.6 W (b) 215.6 W (c) 15.6 W (d) 11.56 W

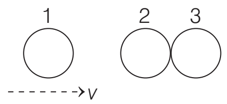
15 Power supplied to a particle of mass 2 kg varies with time as $P = 3\ t^2/2$ W, where t is in second. If velocity of the particle at $t = 0$ is $v = 0$, the velocity of the particle at $t = 2$ s, will be

(a) 1 ms^{-1} (b) 4 ms^{-1} (c) 2 ms^{-1} (d) $2\sqrt{2}$ ms^{-1}

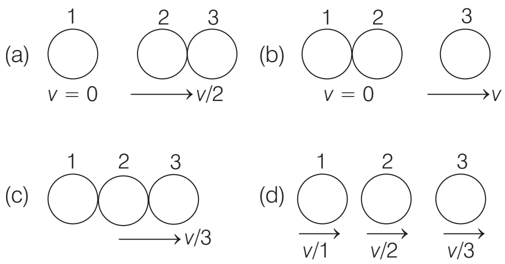
16 A body is moving unidirectionally under the influence of a source of constant power supplying energy. Which of the diagrams as shown in figure correctly shows the displacement-time curve for its motion?



17 Two identical ball bearings in contact with each other and resting on a frictionless table are hit head-on by another ball bearing of the same mass moving initially with a speed v as shown in



If the collision is elastic, which of the following is a possible result after collision?



18 Two balls of masses m_1 and m_2 are separated from each other and a charge is placed between them. The whole system is at rest on the ground. Suddenly, the charge explodes and the masses are pushed apart. The mass m_1 travels a distance S_1 and then it stops. If the coefficient of friction between the balls and the ground are the same, mass m_2 stops after covering the distance

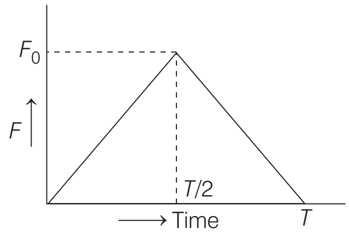
(a) $S_2 = \frac{m_1}{m_2}S_1$ (b) $S_2 = \frac{m_2}{m_1}S_1$

(c) $S_2 = \frac{m_1^2}{m_2^2}S_1$ (d) $S_2 = \frac{m_2^2}{m_1^2}S_1$

19 A shell is fired from a cannon with a velocity v ms^{-1} at an angle θ with the horizontal direction. At the highest point in its path, it explodes into 2 pieces of equal masses. One of the pieces retraces its path to the cannon. The speed in ms^{-1} of the other piece, immediately after the explosion is

(a) $3v\cos\theta$ (b) $2v\cos\theta$ (c) $v\cos\theta$ (d) $\sqrt{\frac{3}{2}}\,v\cos\theta$

20 A particle of mass m moving with a velocity u makes an elastic one dimensional collision with a stationary particle of mass m. They are in contact for a very short interval of time T.



The force of interaction increases from zero to F_0 in $\frac{T}{2}$ and then decreases linearly to zero in further time interval $\frac{T}{2}$.

The magnitude of F_0 is

(a) $\frac{mu}{T}$ (b) $\frac{2mu}{T}$ (c) $\frac{mu}{2T}$ (d) None of these

21 A block of mass 0.50 kg is moving with a speed of 2.00 ms^{-1} on a smooth surface. It strikes another mass of 1.00 kg and then they move together as a single body. The energy loss during the collision is

(a) 0.16 J (b) 1.00 J (c) 0.67 J (d) 0.34 J

22 A ball hits the floor and rebounds after an inelastic collision. In this case

(a) the momentum of the ball just after the collision is the same as that just before the collision

(b) the mechanical energy of the ball remains the same in the collision

(c) the total momentum of the ball and the earth is conserved

(d) total mechanical energy of the ball and the earth is conserved

23 Two particles of masses m_1 and m_2 in projectile motion have velocities $\mathbf{v}_1$ and $\mathbf{v}_2$ respectively at time $t = 0$. They collide at time t_0. Their velocities become $\mathbf{v}'_1$ and $\mathbf{v}'_2$ at time $2t_0$ while still moving in air. The value of $|(m_1 \mathbf{v}'_1 + m_2 \mathbf{v}'_2) - (m_1 \mathbf{v}_1 + m_2 \mathbf{v}_2)|$ is

(a) zero (b) $(m_1 + m_2) g t_0$
(c) $2(m_1 + m_2) g t_0$ (d) $\frac{1}{2}(m_1 + m_2) g t_0$

24 A ball is dropped on the ground from a height of 2m. If the coefficient of restitution is 0.6, the height to which the ball will rebound is

(a) 0.72 m (b) 1.72 m (c) 2.72 m (d) 1 m

25 A body of mass m is accelerated uniformly from rest to a speed v in a time interval T. The instantaneous power delivered to the body as a function of time (t), is given by

(a) $\frac{mv^2}{T^2} t$ (b) $\frac{mv^2}{T^2} t^2$ (c) $\frac{1}{2}\frac{mv^2}{T^2} t$ (d) $\frac{1}{2}\frac{mv^2}{T^2} t^2$

26 A person trying to lose weight by burning fat lifts a mass of 10 kg upto a height of 1 m, 1000 times. Assume that the potential energy lost each time he lowers the mass is dissipated. How much fat will he use up considering the work done only when the weight is lifted up? Fat supplies 3.8×10^7 J of energy per kg which is converted into mechanical energy with a 20% of efficiency rate. (Take, $g = 9.8 \text{ ms}^{-2}$) **→ JEE Main 2016 (Offline)**

(a) 2.45×10^{-3} kg (b) 6.45×10^{-3} kg
(c) 9.89×10^{-3} kg (d) 12.89×10^{-3} kg

27 A time dependent force $F = 6t$ acts on a particle of mass 1 kg. If the particle starts from rest, the work done by the force during the first 1 s will be **→ JEE Main 2017 (Offline)**

(a) 22 J (b) 9 J
(c) 18 J (d) 4.5 J

28 Match the categories of energy given in column I with their formula given in column II and select the correct option from the choices given below.

	Column I		Column II
A.	Spring energy	1.	$\frac{1}{2}mv^2$
B.	Kinetic energy	2.	mgh
C.	Potential energy	3.	$\frac{1}{2}kx^2$

	A	B	C
(a)	3	2	1
(b)	3	1	2
(c)	1	2	3
(d)	2	3	1

Direction (Q. Nos. 29-32) *These question consists of two statements each printed as Statement I and Statement II. While answering these questions you are required to choose any one of the following five responses.*

(a) If both Statement I and Statement II are correct and Statement II is the correct explanation of Statement I.
(b) If both Statement I and Statement II are correct but Statement II is not correct explanation of Statement I.
(c) If Statement I is correct but Statement II is incorrect.
(d) If Statement I is incorrect but Statement II is correct.
(e) If both Statement I and Statement II are incorrect

29 **Statement I** A body cannot have energy without having momentum but it can have momentum without having energy.

Statement II Momentum and energy have same dimensions.

30 **Statement I** When a machine-gun fires n bullets per second with kinetic energy K, then the power of the machine-gun is $P = nK$.

Statement II Power $= \frac{\text{Work}}{\text{Time}} = \frac{nk}{1}$

31 **Statement I** A quick collision between two bodies is more violent than a slow collision; even when the initial and the final velocities are identical.

Statement II The momentum is greater in the first case.

32 **Statement I** A point particle of mass m moving with speed v collides with stationary point particle of mass M. If the maximum energy loss possible is given as $f\left(\frac{1}{2}mv^2\right)$, then $f = \left(\frac{M}{M+m}\right)$.

Statement II Maximum energy loss occurs when the particles get stuck together as a result of the collision. **→ JEE Main 2013**

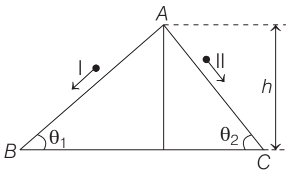
DAY PRACTICE SESSION 2

PROGRESSIVE QUESTIONS EXERCISE

1 A block of mass 0.5 kg has an initial velocity of 10 ms^{-1} while moving down an inclined plane of angle 30°, the coefficient of friction between the block and the inclined surface is 0.2. The velocity of the block, after it covers a distance of 10 m, is

(a) 17 ms^{-1} (b) 13 ms^{-1}
(c) 24 ms^{-1} (d) 8 ms^{-1}

2 Two inclined frictionless tracks, one gradual and the other steep meet at A from where two stones are allowed to slide down from rest, one on each track as shown in figure.



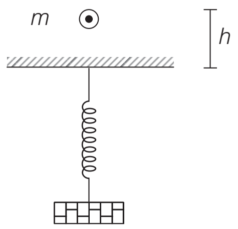
Which of the following statement is correct ?

(a) Both the stones reach the bottom at the same time but not with the same speed
(b) Both the stones reach the bottom with the same speed and stone I reaches the bottom earlier than stone II
(c) Both the stones reach the bottom with the same speed and stone II reaches the bottom earlier than stone I
(d) Both the stones reach the bottom at different times and with different speeds

3 Suppose the average mass of raindrops is 3×10^{-5} kg and their average terminal velocity 9 ms^{-1}. Then the energy transferred by rain to each square metre of the surface at a place which receives 100 cm of rain in a year is

(a) 302×10^2 J (b) 102×10^5 J
(c) 4.05×10^4 J (d) 9.2×10^3 J

4 A ball of mass m is dropped from a height h on a massless platform fixed at the top of a vertical spring as shown below. The platform is depressed by a distance x. What will be the value of the spring constant?



(a) $2mg/x$ (b) $2mgh/x$
(c) $2mg(h+x)/x^2$ (d) $2mg(h) + 2mghx/x^2$

5 A force $\mathbf{F} = -k(y\,\mathbf{i} + x\,\mathbf{j})$, acts on a particle moving in the xy-plane. Starting from the origin, the particle is taken along the positive x-axis to the point $(a, 0)$ and is then taken parallel to the y-axis to the point (a, a). The total work done by the force is

(a) $-2ka^2$ (b) $2ka^2$ (c) $-ka^2$ (d) ka^2

6 A uniform chain of length 2 m is kept on a table such that a length of 60 cm hangs freely from the edge of the table. The total mass of the chain is 4 kg. What is the work done in pulling the entire chain on the table?

(a) 7.2 J (b) 3.6 J (c) 120 J (d) 1200 J

7 A 70 kg man leaps vertically into the air from a crouching position. To take the leap the man pushes the ground with a constant force F to raise himself.

The centre of gravity rises by 0.5 m before he leaps. After the leap the centre of gravity rises by another 1 m. The maximum power delivered by the muscles is (take, $g = 10\ ms^{-2}$). **➞ JEE Main (Online) 2013**

(a) 6.26×10^3 W at the start (b) 6.26×10^3 W at take off
(c) 6.26×10^4 W at the start (d) 6.26×10^4 W at take off

8 If two springs S_1 and S_2 of force constants k_1 and k_2, respectively are stretched by the same force, it is found that more work is done on spring S_1 than on spring S_2. The correct option is

(a) $w_1 = w_2$ (b) $k_1 < k_2$
(c) $k_1 > k_2$ (d) None of these

9 At time $t = 0$ s particle starts moving along the x-axis. If its kinetic energy increases uniformly with time t, the net force acting on it must be proportional to

(a) $\sqrt{t}$ (b) constant (c) t (d) $\frac{1}{\sqrt{t}}$

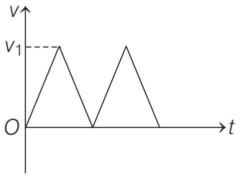
10 It is found that, if a neutron suffers an elastic collinear collision with deuterium at rest, fractional loss of its energy is P_d; while for its similar collision with carbon nucleus at rest, fractional loss of energy is P_c. The values of P_d and P_c are respectively

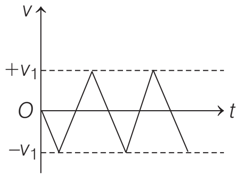
(a) (.89, .28) (b) (.28, .89)
(c) (0, 0) (d) (0, 1)

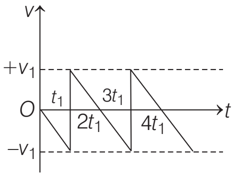
11 A particle of mass m moving in the x-direction with speed $2v$ is hit by another particle of mass $2m$ moving in the y-direction with speed v. If the collision is perfectly inelastic, the percentage loss in the energy during the collision is close to **➞ JEE Main 2015**

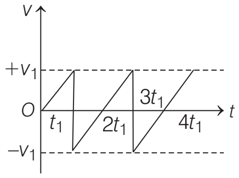
(a) 44% (b) 50% (c) 56% (d) 62%

12 Consider a rubber ball freely falling from a height $h = 4.9$ m onto a horizontal elastic plate. Assume that the duration of collision is negligible and the collision with the plate is totally elastic. Then, the velocity as a function of time the height as function of time will be

(a) 

(b) 

(c) 

(d) 

13 A body of mass $m = 10^{-2}$ kg is moving in a medium and experiences a frictional force $F = -kv^2$. Its initial speed is $v_0 = 10\,\text{ms}^{-1}$. If after 10 s, its energy is $\frac{1}{8}mv_0^2$, the value of k will be **➞ JEE Main 2017 (Offline)**

(a) $10^3\,\text{kgs}^{-1}$ (b) $10^{-4}\,\text{kgm}^{-1}$
(c) $10^{-1}\,\text{kgm}^{-1}\text{s}^{-1}$ (d) $10^{-3}\,\text{kgm}^{-1}$

14 A uniform chain of length l and weight w is hanging from its ends A and B which are close together. At a given instant end B is released. The tension at A when B has fallen a distance $x < \frac{l}{2}$ is

(a) $\frac{w}{2}\left[\frac{3x}{l} - 2\right]$ (b) $\frac{w}{2}\left[3 - \frac{3x}{4}\right]$
(c) $\frac{w}{2}\cdot\left[1 + \frac{3x}{l}\right]$ (d) $\frac{w}{2}\left[\frac{3x}{l} + 4\right]$

15 A body is moved along a straight line by a machine delivering a constant power. The distance moved by the body in time t is proportional to

(a) $t^{3/4}$ (b) $t^{3/2}$
(c) $t^{1/4}$ (d) $t^{1/2}$

16 In a collinear collision, a particle with an initial speed v_0 strikes a stationary particle of the same mass. If the final total kinetic energy is 50% greater than the original kinetic energy, the magnitude of the relative velocity between the two particles after collision, is **➞ JEE Main 2018**

(a) $\frac{v_0}{4}$ (b) $\sqrt{2}\,v_0$
(c) $\frac{v_0}{2}$ (d) $\frac{v_0}{\sqrt{2}}$

17 A mass m moves with a velocity v and collides inelastically with another identical mass. After collision, the 1st mass moves with velocity $\frac{v}{\sqrt{3}}$ in a direction perpendicular to the initial direction of motion. Find the speed of the second mass after collision.

(a) v (b) $\sqrt{3}\,v$ (c) $\frac{2}{\sqrt{3}}v$ (d) $\frac{v}{\sqrt{3}}$

18 The potential energy function for the force between two atoms in a diatomic molecule is approximately given by $U(x) = \frac{a}{x^{12}} - \frac{b}{x^6}$, where a and b are constants and x is the distance between the atoms. If the dissociation energy of the molecule is $D = [U(x = \infty) - U_{\text{at equilibrium}}]$, D is

(a) $\frac{b^2}{2a}$ (b) $\frac{b^2}{12a}$ (c) $\frac{b^2}{4a}$ (d) $\frac{b^2}{6a}$

19 Under the action of force, 3 kg body moves such that its position x as a function of time t is given by $x = \frac{t^3}{3}$, where x in meter and t in second. Then, work done by the force in the first 2 s is

(a) 8 J (b) 16 J (c) 24 J (d) 30 J

20 When a rubber band is stretched by a distance x, it exerts a restoring force of magnitude $F = ax + bx^2$, where, a and b are constants. The work done in stretching the unstretched rubber band by L is **➞ JEE Main 2014**

(a) $aL^2 + bL^3$ (b) $\frac{1}{2}(aL^2 + bL^3)$
(c) $\frac{aL^2}{2} + \frac{bL^3}{3}$ (d) $\frac{1}{2}\left(\frac{aL^2}{2} + \frac{bL^3}{3}\right)$

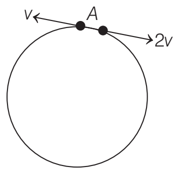
21 A spring gun having a spring of spring constant k is placed at a height h. A ball of mass m is placed in its barrel and compressed by a distance x. Where shall we place a box on the ground, so that the ball lands in the box?

(a) $\frac{kh}{mg}x$ (b) $\sqrt{\frac{2kh}{mg}}x$ (c) $\sqrt{\frac{kh}{2mg}}x$ (d) $\frac{kh}{2mg}$

22 A particle of mass m is projected from the ground with an initial speed u_0 at an angle α with the horizontal. At the highest point of its trajectory, it makes a completely inelastic collision with another identical particle, which was thrown vertically upward from the ground with the same initial speed u_0. The angle that the composite system makes with the horizontal immediately after the collision is

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{4} + \alpha$ (c) $\frac{\pi}{4} - \alpha$ (d) $\frac{\pi}{2}$

23 Two small particles of equal masses start moving in opposite directions from a point A in a horizontal circular orbit. Their tangential velocities are v and $2v$ respectively, as shown in the figure. Between collisions, the particles move with constant speeds. After making how many elastic collisions, other than that at A, these two particles will again reach the point A?



(a) 4 (b) 3 (c) 2 (d) 1

Direction (Q. Nos. 24-25) *Each of these questions contains two statements : Statement* I *(Assertion) and Statement* II *(Reason). Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c), (d) given below*

(a) Statement I is correct, Statement II is correct; Statement II is the correct explanation for Statement I

(b) Statement I is correct, Statement II is correct; Statement II is not the correct explanation for Statement I

(c) Statement I is correct; Statement II is incorrect

(d) Statement I is incorrect; Statement II is correct

24 **Statement I** Two bodies of different masses have the same momentum, and their kinetic energies are in the inverse ratio of their masses.

Statement II Kinetic energy of body is given by the relation.

$$\text{KE} = \frac{1}{2}mv^2$$

25 **Statement I** An object is displaced from point $A(2\text{m}, 3\text{ m}, 4\text{ m})$ to a point $B(1\text{ m}, 2\text{ m}, 3\text{ m})$ under a constant force $\mathbf{F} = (2\mathbf{i} + 3\mathbf{j} + 4\mathbf{k})$N. The work done by the force in this process is –9 J.

Statement II Work done by a force, an object can be given by the relation,

$$W = \int_{r_1}^{r_2} \mathbf{F} \cdot d\mathbf{r} \quad \text{or} \quad W = \mathbf{F} \cdot \mathbf{s}$$

ANSWERS

SESSION 1	**1** (c)	**2** (c)	**3** (a)	**4** (b)	**5** (c)	**6** (a)	**7** (d)	**8** (c)	**9** (a)	**10** (a)
	11 (d)	**12** (b)	**13** (a)	**14** (a)	**15** (c)	**16** (b)	**17** (b)	**18** (c)	**19** (a)	**20** (b)
	21 (c)	**22** (c)	**23** (c)	**24** (a)	**25** (a)	**26** (d)	**27** (d)	**28** (b)	**29** (e)	**30** (a)
	31 (a)	**32** (a)								
SESSION 2	**1** (b)	**2** (c)	**3** (c)	**4** (c)	**5** (c)	**6** (b)	**7** (b)	**8** (b)	**9** (d)	**10** (a)
	11 (c)	**12** (c)	**13** (b)	**14** (c)	**15** (b)	**16** (b)	**17** (c)	**18** (c)	**19** (c)	**20** (c)
	21 (b)	**22** (a)	**23** (c)	**24** (a)	**25** (a)					

Hints and Explanations

SESSION 1

1 Resultant force,

$$F = \sqrt{3^2 + 4^2} = 5\text{N}$$

$$a = \frac{F}{m} = \frac{5}{2} = 2.5\,\text{m/s}^2$$

$$u = 0, t = 20\,\text{s}$$

$$\therefore \quad s = ut + \frac{1}{2}at^2$$

$$= 0 \times 20 + \frac{1}{2} \times 2.5 \times (20)^2 = 500\,\text{m}$$

Hence, work done

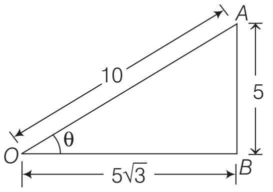
$$W = F \cdot s = 5 \times 500 = 2500\,\text{J}$$

2 Work done in the round trip = total work done against friction while moving up and down the plane

$$= 2(\mu\, mg \cos\theta) \times s$$

$$= 2(0.1 \times 0.5 \times 10 \times \frac{\sqrt{3}}{2} \times 10)$$

$$= 5\sqrt{3}\ \text{J} \quad \left[\because \cos\theta = \frac{5\sqrt{3}}{10} = \frac{\sqrt{3}}{2}\right]$$



3 Work done among horizontal is

$$dW = F \cdot dx$$

or $\quad dW = (9 - x^2) \cdot dx$

For total work done, integrate both sides with proper limit

$$\int_0^W dW = \int_0^3 (9 - x^2)dx$$

or $\quad W = \left[9x - \frac{x^3}{3}\right]_0^3$

$$= \left(27 - \frac{27}{3}\right) - (0 - 0) = 18\,\text{J}$$

$\therefore$ Maximum work done is 18 J.

4 Since, $\mathbf{F}$ = constant,

we can also use

$$W = \mathbf{F} \cdot \mathbf{s}$$

Here, $\mathbf{s} = \mathbf{r}_f - \mathbf{r}_i = (\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}) - (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}})$

$$= (-\hat{\mathbf{i}} - \hat{\mathbf{j}} - \hat{\mathbf{k}})$$

$$\therefore \quad W = (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}}) \cdot (-\hat{\mathbf{i}} - \hat{\mathbf{j}} - \hat{\mathbf{k}})$$

$$= -2 - 3 - 4 = -9\,\text{J}$$

5 If v' is the final velocity of the wagon, then applying the principle of conservation of linear momentum, we get

$$5 \times 10^3 \times 1.2 = (5 \times 10^3 + 10^3) \times v'$$

$$v' = 1\,\text{ms}^{-1}$$

Change in KE

$$= \frac{1}{2}(6 \times 10^3) \times 1^2 - \frac{1}{2}(5 \times 10^3)(1.2)^2$$

$= -600$ J [minus sign for the loss in kinetic energy]

6 $a = -kx$ or $\dfrac{vdv}{dx} = -kx$

$$\Rightarrow \quad v\,dv = -kx dx$$

Let the velocity change from v_0 to v.

$$\Rightarrow \quad \int_{v_0}^{v} dv = -\int_0^x k\,x dx$$

$$\Rightarrow \quad \frac{v^2 - v_0^2}{2} = -\frac{k\,x^2}{2}$$

$$\Rightarrow \quad m\left(\frac{v^2 - v_0^2}{2}\right) = -\frac{mk\,x^2}{2}$$

$$\Rightarrow \quad \Delta k \propto x^2$$

[Δk is loss in kinetic energy]

7 As, the earth moves ones around the sun in its elliptical orbit, its KE, is maximum when it is closest to the sun and minimum when it is farthest from the sun. Also, KE of the earth is never zero during its motion choice (d) is correct.

8 $\therefore$ Force $= -\dfrac{dU}{dr}$

$$\Rightarrow \quad F = -\frac{d}{dr}\left(\frac{-k}{2r^2}\right) = -\frac{k}{r^3}$$

As particle is on circular path, this force must be centripetal force.

$$\Rightarrow \quad |F| = \frac{mv^2}{r}$$

So, $\quad \dfrac{k}{r^3} = \dfrac{mv^2}{r} \quad \Rightarrow \quad \dfrac{1}{2}mv^2 = \dfrac{k}{2r^2}$

$\therefore$ Total energy of particle = KE + PE

$$= \frac{k}{2r^2} - \frac{k}{2r^2}$$

$= 0$

Total energy = 0

9 $V(x) = \left(\dfrac{x^4}{4} - \dfrac{x^2}{2}\right)$ J

For minimum value of V, $\dfrac{dV}{dx} = 0$

$$\Rightarrow \quad \frac{4x^3}{4} - \frac{2x}{2} = 0$$

$$\Rightarrow \quad x = 0,\ x = \pm 1$$

So, $\quad V_{\min}(x = \pm 1) = \dfrac{1}{4} - \dfrac{1}{2} = \dfrac{-1}{4}$ J

Now, $K_{\max} + V_{\min}$
= total mechanical energy

$$\Rightarrow \quad K_{\max} = \left(\frac{1}{4}\right) + 2$$

or $\quad K_{\max} = \dfrac{9}{4}$

or $\quad \dfrac{mv^2}{2} = \dfrac{9}{4}$

or $\quad v = \dfrac{3}{\sqrt{2}}\,\text{ms}^{-1}$

10 From $\quad F = -\dfrac{dU}{dx}$

$$\int_0^{U(x)} dU = -\int_0^x F\,dx = -\int_0^x (kx)dx$$

$$\therefore \quad U(x) = -\frac{kx^2}{2} \text{ as } U(0) = 0$$

11 According to the work-energy theorem,

$$W = \Delta K$$

***Case* I**

$$-F \times 3 = \frac{1}{2}m\left(\frac{v_0}{2}\right)^2 - \frac{1}{2}mv_0^2$$

where, F is the resistive force and v_0 is the initial speed.

***Case* II** Let, the further distance travelled by the bullet before coming to rest is s.

$$\therefore \quad -F(3 + s) = K_f - K_i = -\frac{1}{2}mv_0^2$$

$$\Rightarrow \quad -\frac{1}{8}mv_0^2(3 + s) = -\frac{1}{2}mv_0^2$$

or $\quad \dfrac{1}{4}(3 + s) = 1$

or $\quad \dfrac{3}{4} + \dfrac{s}{4} = 1$

or $\quad s = 1$ cm

12 From the work-energy theorem, kinetic energy of the block at a distance x is

$$K = \int_0^x F\,dx = \int_0^x (4 - x^2)dx = 4x - \frac{x^3}{3}$$

For kinetic energy to be maximum,

$$\frac{dK}{dx} = 0$$

$$\frac{d}{dx}\left(4x - \frac{x^3}{3}\right) = 0$$

$$4 - x^2 = 0 \text{ or } x = \pm 2\,\text{m}$$

At $\quad x = +2\text{m}, \dfrac{d^2K}{dx^2}$ = negative

i.e. Kinetic energy is maximum.

$$K_{\max} = 4x - \frac{x^3}{3}$$

$$= 4(2) - \frac{2^3}{3} = 5.33\,\text{J}$$

13 According to the conservation of energy,

$$\frac{1}{2}kL^2 = \frac{1}{2}Mv^2$$

$$\Rightarrow \quad kL^2 = \frac{(Mv)^2}{M}$$

or $\quad MkL^2 = p^2 \quad [\because p = Mv]$

$$\Rightarrow \quad p = L\sqrt{Mk}$$

14 $v = r\omega = r\frac{2\pi}{t} = \frac{1}{4} \times \frac{2\pi}{2} = \frac{\pi}{4}\ \text{ms}^{-1}$

$P = F \times v = (15 \times 9.8) \times \frac{\pi}{4} = 115.6\ \text{W}$

15 From the work-energy theorem,

$$\Delta KE = W_{net}$$

$$K_f - K_i = \int P\,dt$$

$$\frac{1}{2}mv^2 - 0 = \int_0^2 \left(\frac{3}{2}t^2\right)dt$$

$$\frac{1}{2}(2)v^2 = \frac{3}{2}\int_0^2 t^2 dt$$

$$= \frac{3}{2} \times \left[\frac{t^3}{3}\right]_0^2 = 4$$

$\Rightarrow \quad v = 2\ \text{ms}^{-1}$

16 Here, $P = [\text{ML}^2\ \text{T}^{-3}]$ = constant

As mass M of body is fixed

$[\text{L}^2\ \text{T}^{-3}]$ = constant

$\Rightarrow \quad \frac{[\text{L}^2]}{[\text{T}^3]}$ = constant

$\Rightarrow \quad [\text{L}] \propto [\text{T}^{3/2}]$

$\Rightarrow$ Displacement $\propto t^{3/2}$

17 As the ball bearings are identical, their masses are equal. In elastic collision, their velocities are interchanged. In collision between 1 and 2; velocity of 1 becomes zero and velocity of 2 becomes v. In collision between 2 and 3, velocity of 2 becomes zero and velocity of 3 becomes v.

18 From the conservation of momentum, we get

$$m_1 v_1 = m_2 v_2 \quad \text{...(i)}$$

also, $\frac{1}{2}m_1 v_1^2 = f_1 S_1 = \mu m_1 g S_1$ ...(ii)

and $\frac{1}{2}m_2 v_2^2 = f_2 S_2 = \mu m_2 g S_2$ ...(iii)

where, μ = coefficient of friction

On dividing Eq. (ii) by Eq. (iii), we get

$$\frac{m_1 v_1^2}{m_2 v_2^2} = \frac{m_1 S_1}{m_2 S_2} \quad \text{...(iv)}$$

Using Eqs. (i) and (iv), we get

$$\frac{v_1}{v_2} = \frac{m_1 S_1}{m_2 S_2}$$

or $\frac{m_2}{m_1} = \frac{m_1 S_1}{m_2 S_2} \Rightarrow S_2 = \frac{m_1^2}{m_2^2} S_1$

19 Velocity at the highest point = $v\cos\theta$

Applying the principle of conservation of linear momentum, we get

$$2m(v\cos\theta) = m(-v\cos\theta) + mv'$$

$$v' = 3\,v\cos\theta$$

20 The collision will cause an exchange of velocities. The change in momentum of any particle = mu, which is equal to the impulse = area under the force-time graph

$$mu = \frac{1}{2}F_0 \times T$$

$\Rightarrow \quad F_0 = 2\frac{mu}{T}$

21 $m_1 u_1 + m_2 u_2 = (m_1 + m_2)v$

$\Rightarrow \quad v = 2/3\ \text{ms}^{-1}$

Energy loss

$= \frac{1}{2}(0.5) \times (2)^2 - \frac{1}{2}(1.5) \times \left(\frac{2}{3}\right)^2$

$= 0.67\ \text{J}$

22 In an inelastic collision only momentum of the system may remain conserved. Some energy can be lost in the form of heat, sound, etc.

23 $|(m_1 \mathbf{v}_1' + m_2 \mathbf{v}_2') - (m_1 \mathbf{v}_1 + m_2 \mathbf{v}_2)|$

= |change in momentum of the two particles|

= |External force on the system| $\times$ time interval

$= (m_1 + m_2)\,g\,(2t_0) = 2(m_1 + m_2)\,g t_0$

24 Let v is the final velocity of ball on reaching the ground, then

$$v = \sqrt{2gh} = \sqrt{2 \times g \times 2}$$

or $\quad v = 2\sqrt{g}$

For upward motion

$$u = 2\sqrt{g} \times e \quad \text{and} \quad v = 0$$

$\therefore$ Height upto which the ball will rebound is

$$H = \frac{u^2}{2g} = \frac{(2\sqrt{g} \times e)^2}{2g}$$

$$= \frac{4g \times e^2}{2g} = 2 \times 0.6 \times 0.6 = 0.72\ \text{m}$$

25 $F = ma = \frac{mv}{T} \quad \left(\because a = \frac{v - 0}{T}\right)$

Instantaneous power = $Fv = mav$

$$= \frac{mv}{T}at$$

$$= \frac{mv}{T}\frac{v}{T}t = \frac{mv^2}{T^2}t$$

26 Given, potential energy burnt by lifting weight = $mgh = 10 \times 9.8 \times 1 \times 1000$

$= 9.8 \times 10^4$

If mass lost by a person be m, then

energy dissipated = $m \times \frac{2}{10} \times 3.8 \times 10^7$ J

$\Rightarrow \quad 9.8 \times 10^4 = m \times \frac{1}{5} \times 3.8 \times 10^7$

$\Rightarrow \quad m = \frac{5}{3.8} \times 10^{-3} \times 9.8$

$= 12.89 \times 10^{-3}$ kg

27 From Newton's second law,

$$\frac{\Delta p}{\Delta t} = F \Rightarrow \Delta p = F\Delta t$$

$\therefore \quad p = \int dp = \int_0^1 F\,dt$

$\Rightarrow \quad p = \int_0^1 6t\,dt = 3\ \text{kg}\left(\frac{\text{m}}{\text{s}}\right)$

Also, change in kinetic energy

$$\Delta k = \frac{\Delta p^2}{2m} = \frac{3^2}{2 \times 1} = 4.5$$

From work-energy theorem,

Work done = change in kinetic energy

So, work done = Δk = 4.5 J

28 Kinetic energy = $\frac{1}{2}mv^2$

Potential energy = mgh

Spring energy = $\frac{1}{2}kx^2$

29 We know that a body may not have momentum but may have potential energy by virtue of its position as in case of a stretched or a compressed spring. But when the body does not contain energy then its kinetic energy is zero hence, its momentum is also zero.

Dimensions of momentum

$$(mv) = [\text{MLT}^{-1}]$$

Dimensions of energy

$$\left(\frac{1}{2}mv^2\right) = [\text{ML}^2\text{T}^{-2}]$$

30 $\therefore$ Power = $\frac{\text{Work}}{\text{Time}} = \frac{n \times K}{1} = \frac{nK}{1}$

31 Momentum $p = mv$ or $p \propto v$,

i.e. momentum is directly proportional to the velocity, so the momentum is greater in a quicker collision between two bodies than in a slower one. Hence, due to greater momentum, quicker collision between two bodies will be more violent even if the initial and the final velocities are identical.

32 Energy $E = \frac{p^2}{2m}$, where p is momentum,

m is the mass moving of the particle. Maximum energy loss occurs when the particles get stuck together as a result of the collision.

Maximum energy loss (ΔE)

$$= \frac{p^2}{2m} - \frac{p^2}{2(m + M)}$$

where, $(m + M)$ is the resultant mass when the particles get stuck.

$$\Delta E = \frac{p^2}{2m}\left[1 - \frac{m}{m + M}\right] = \frac{p^2}{2m}\left[\frac{M}{m + M}\right]$$

Also, $p = mv$

$$\therefore \quad \Delta E = \frac{m^2 v^2}{2m}\left[\frac{M}{m + M}\right]$$

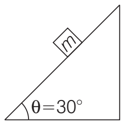
$$= \frac{mv^2}{2}\left[\frac{M}{m + M}\right]$$

Comparing the expression with

$$\Delta E = f\left(\frac{1}{2}mv^2\right), f = \frac{M}{m + M}$$

SESSION 2

1 Here, $m = 0.5$ kg, $u = 10\ \text{ms}^{-1}$, $\theta = 30°$, $\mu = 0.2$, $s = 10$ m



Acceleration down the plane,

$$a = g(\sin\theta - \mu\cos\theta)$$
$$= 10\ (\sin 30° - 0.2\cos 30°)$$
$$= 3.268\ \text{ms}^{-2}$$

From second equation of motion,

$$v^2 = u^2 + 2as$$
$$= 10^2 + 2\ (3.268) \times 10$$
$$= 165.36$$
$$v = \sqrt{165.36}$$
$$\approx 13\ \text{ms}^{-1}$$

2 As, both surfaces I and II are frictionless and two stones slide from rest from the same height, therefore, both the stones reach the bottom with same speed $\left(\frac{1}{2}mv^2 = mgh\right)$.

As acceleration down plane II is larger ($a_2 = g\sin\theta_2$ is greater than $a_1 = g\sin\theta_1$), therefore, stone II reaches the bottom earlier than stone I.

3 Given, average mass of rain drop $(m) = 3.0 \times 10^{-5}$ kg

Average terminal velocity $= (v)$
$= 9\ \text{ms}^{-1}$

Height $(h) = 100$ cm $= 1$m

Density of water $(\rho) = 10^3\ \text{kgm}^{-3}$

Area of the surface $(A) = 1\text{m}^2 = A \times h$
$= 1 \times 1$
$= 1\text{m}^3$

Mass of the water due to rain (M)
$=$ Volume $\times$ density
$= V \times \rho = 1 \times 10^3$
$= 10^3$ kg

$\therefore$ Energy transferred to the surface

$$= \frac{1}{2}mv^2 = \frac{1}{2} \times 10^3 \times (9)^2$$
$$= 40.5 \times 10^3\ \text{J}$$
$$= 4.05 \times 10^4\ \text{J}$$

4 Here, $mg(h + x) = \frac{1}{2}kx^2$

$$\Rightarrow \quad kx^2 = 2mg(h + x)$$
$$\Rightarrow \quad k = \frac{2mg(h + x)}{x^2}$$

5 $W_1 = \int_0^a \mathbf{F}d\mathbf{x} = \int_0^a -k(y\hat{\mathbf{i}} + x\hat{\mathbf{j}})\hat{\mathbf{i}}\,dx$

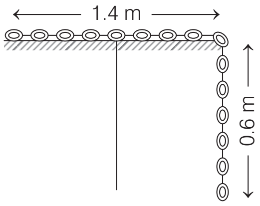
$$= \int_0^a -k(0\hat{\mathbf{i}} + x\hat{\mathbf{j}})\hat{\mathbf{i}}\,dx = \text{zero}$$

$$W_2 = \int_0^a \mathbf{F}\cdot d\,\mathbf{y} = \int_0^a -k(y\hat{\mathbf{i}} + x\hat{\mathbf{j}})\,\hat{\mathbf{j}}\,dy$$
$$= \int_0^a -k(a\hat{\mathbf{i}} + a\hat{\mathbf{j}})\hat{\mathbf{j}}\,dy$$
$$= -ka\int_0^a dy = -ka^2$$

Total work done,
$W = W_1 + W_2 = 0 - ka^2 = -ka^2$

6 Mass per unit length $= \frac{M}{L}$

$= \frac{4}{2} = 2\ \text{kg m}^{-1}$



The mass of 0.6 m of chain $= 0.6 \times 2$
$= 1.2$ kg

The height of the centre of mass of the hanging part

$$h = \frac{0.6 + 0}{2} = 0.3\ \text{m}$$

Hence, work done in pulling the chain on the table $=$ work done against the force of gravity

i.e. $W = mgh = 1.2 \times 10 \times 0.3 = 3.6$ J

7 As, $\quad P = F \cdot v$

So, $\quad \frac{dP}{dt} = F \cdot \frac{dv}{dt}$

To deliver the maximum power $\frac{dP}{dt} = 0$,

which gives

$$P_{\text{max}} = 6.26 \times 10^3\ \text{W}$$

8 As no relation between k_1 and k_2 is given in the question.
Then, for same force

$$W = Fx = F\frac{F}{k} = \frac{F^2}{k} \Rightarrow W \propto \frac{1}{k}$$

i.e. $W_1 > W_2 \Rightarrow k_1 < k_2$

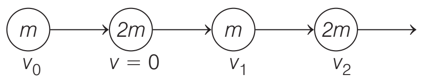
9 Given, $\frac{dk}{dt} =$ constant $\Rightarrow k \propto t$

$$\Rightarrow \quad v \propto \sqrt{t}$$

Also, $\quad p = Fv = \frac{dk}{dt} =$ constant

$$\Rightarrow \quad F \propto \frac{1}{v} \quad \Rightarrow \quad F \propto \frac{1}{\sqrt{t}}$$

10 Neutron-Deuterium collision;



Momentum conservation gives;

$$mv_0 = mv_1 + 2mv_2$$
$$\Rightarrow \quad v_0 = v_1 + 2v_2 \quad \ldots\text{(i)}$$

Collision given is elastic .

So, coefficient of restitution, $e = 1$

$$\therefore \quad e = 1 = \frac{\text{Velocity of separation}}{\text{Velocity of approach}}$$
$$\Rightarrow \quad 1 = \frac{v_2 - v_1}{v_0 - 0}$$
$$\Rightarrow v_0 = v_2 - v_1 \quad \ldots\text{(ii)}$$

On adding Eqs. (i) and (ii), we get

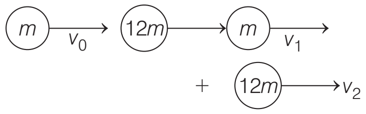
$$2v_0 = 3v_2 \Rightarrow \frac{2v_0}{3} = v_2$$

So, from Eq. (i), we get

$$v_1 = v_0 - 2v_2 = v_0 - \frac{4v_0}{3}$$
$$\Rightarrow \quad v_1 = -\frac{v_0}{3}$$

Fractional loss of energy of neutron

$$= \left(\frac{-K_f + K_i}{K_i}\right)_{\text{for neutron}}$$
$$= \frac{-\frac{1}{2}mv_1^2 + \frac{1}{2}mv_0^2}{\frac{1}{2}mv_0^2} = \frac{-\frac{v_0^2}{9} + v_0^2}{v_0^2}$$
$$= \left(-\frac{1}{9} + 1\right) = \frac{8}{9} = 0.8\bar{8}$$
$$= 0.89$$



Similarly, for neutron-carbon atom collision;

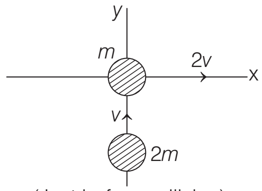
Momentum conservation gives;

$$v_0 = v_1 + 12v_2 \text{ and } e = 1$$
$$\Rightarrow \quad v_0 = v_2 - v_1$$
$$\text{So,} \quad v_1 = \frac{11}{13}v_0$$

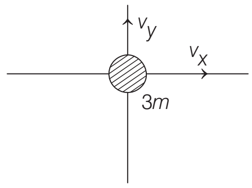
$\therefore$ Loss of energy $= \left(-\frac{121}{169} + 1\right) = 0.28$

So, $P_d = 0.89$ and $P_c = 0.28$

11 Conservation of linear momentum can be applied but energy is not conserved. Consider the movement of two particles as shown below :



(Just before collision)



(Just after collision)

According to conservation of linear momentum in x-direction we have

$(p_1)x = (p_1)x$ or $2mv = (2m + m)v_x$

or $v_2 = \frac{2}{3}v$

As, conserving linear momentum in y-direction, we get

$$(p_i)y = (p_t)y$$

or $\quad 2\,mv = (2m + m)v_x$

or $\quad v_y = \frac{2}{3}v$

Initial kinetic energy of the two particles system is

$$E = \frac{1}{2}m(2v)^2 + \frac{1}{2}(2m)(v)^2$$

$$= \frac{1}{2}\times 4mv^2 + \frac{1}{2}\times 2mv^2$$

$$= 2\,mv^2 + mv^2 = 3mv^2$$

Final energy of the combined two particles system is

$$E_t = \frac{1}{2}(3m)(v_x^2 + v_y^2)$$

$$= \frac{1}{2}(3m)\left[\frac{4v^2}{9} + \frac{4v^2}{9}\right]$$

$$= \frac{3m}{2}\left[\frac{8v^2}{9}\right] = \frac{4mv^2}{3}$$

Loss in the energy,

$$\Delta E = E_i - E_f$$

$$= mv^2\left[3 - \frac{4}{3}\right] = \frac{5}{3}mv^2$$

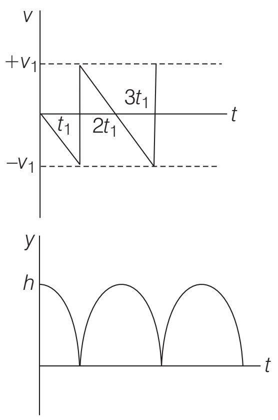
Percentage loss in the energy during the collision

$$\frac{\Delta E}{E_i}\times 100 = \frac{\frac{5}{3}mv^2}{3mv^2}\times 100$$

$$= \frac{5}{9}\times 100 = 56\%$$

12 As we know that for vertical motion,

$$h = \frac{1}{2}gt^2 \qquad \text{[parabolic]}$$



$v = -gt$ and after the collision,

$v = gt$ (straight line). $\quad [\because v = u + gt]$

Collision is perfectly elastic, then ball reaches to same height again and again with same velocity.

Hence, option (c) is true.

13 Given force, $F = -kv^2$

$\therefore$ Acceleration, $a = \frac{-k}{m}v^2$

or $\quad \frac{dv}{dt} = \frac{-k}{m}v^2$

$\Rightarrow \quad \frac{dv}{v^2} = -\frac{k}{m}dt$

Now, with limits, we have

$$\int_{10}^{v}\frac{dv}{v^2} = -\frac{k}{m}\int_0^t dt$$

$$\Rightarrow \quad \left(-\frac{1}{v}\right)_{10}^{v} = -\frac{k}{m}t$$

$$\Rightarrow \quad \frac{1}{v} = 0.1 + \frac{kt}{m}$$

$$\Rightarrow \quad v = \frac{1}{0.1 + \frac{kt}{m}} = \frac{1}{0.1 + 1000k}$$

$$\Rightarrow \quad \frac{1}{2}\times m\times v^2 = \frac{1}{8}mv_0^2$$

$$\Rightarrow \quad v = \frac{v_0}{2} = 5$$

$$\Rightarrow \quad \frac{1}{0.1 + 1000k} = 5$$

$$\Rightarrow \quad 1 = 0.5 + 5000k$$

$$\Rightarrow \quad k = \frac{0.5}{5000}$$

$$\Rightarrow \quad k = 10^{-4}\ \text{kg/m}$$

14 Mass per unit length,

$$\lambda = \frac{m}{l} = \frac{w}{lg}$$

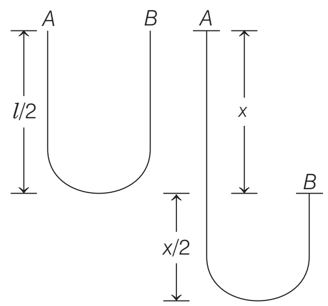
Velocity, $\quad v^2 = 2g\left(\frac{x}{2}\right)$

or $\quad v = \sqrt{gx}$

Change in momentum when an element dx falls is $\frac{w}{lg}\sqrt{gx}\,dx$

$$F = \frac{dp}{dt} = \frac{w}{lg}\sqrt{gx}\frac{dx}{dt} = \frac{w}{l}x$$

$$\left(\because \frac{dx}{dt} = \sqrt{gx}\right)$$



Tension at A

= Weight of half the chain

+ Weight of $\frac{x}{2}$ length + F

$$= \frac{w}{2} + \frac{wx}{2l} + \frac{wx}{l}$$

$$= \frac{w}{2}\left[1 + \frac{3x}{l}\right]$$

15 P = constant

$\Rightarrow \quad Fv = P \quad [\because P = \text{force}\times\text{velocity}]$

$\Rightarrow \quad Ma\times v = P$

$\Rightarrow \quad va = \frac{P}{M}$

$\Rightarrow v\times\left[\frac{v\,dv}{ds}\right] = \frac{P}{M} \qquad \left[\because a = \frac{v\,dv}{ds}\right]$

$\Rightarrow \quad \int_0^v v^2dv = \int_0^s \frac{P}{M}ds$

[Assuming at $t = 0$ it starts from rest, i.e. from $s = 0$]

or $\quad \frac{v^3}{3} = \frac{P}{M}s$

$\Rightarrow \quad v = \left(\frac{3P}{M}\right)^{1/3}s^{1/3}$

$\Rightarrow \quad \frac{ds}{dt} = ks^{1/3} \quad \left[k = \left(\frac{3P}{M}\right)^{1/3}\right]$

$\Rightarrow \quad \int_0^s \frac{ds}{s^{1/3}} = \int_0^t k\,dt$

or $\quad \frac{s^{2/3}}{2/3} = kt$

$\Rightarrow \quad s^{2/3} = \frac{2}{3}kt$

or $\quad s = \left(\frac{2}{3}k\right)^{3/2}\times t^{3/2}$

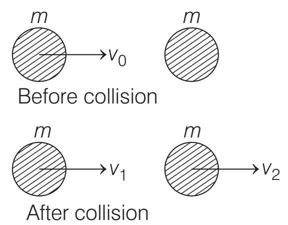
or $\quad s \propto t^{3/2}$

16 Momentum is conserved in all type of collisions,

Final kinetic energy is 50% more than initial kinetic energy

$$\Rightarrow \quad \frac{1}{2}mv_2^2 + \frac{1}{2}mv_1^2$$

$$= \frac{150}{100}\times\frac{1}{2}mv_0^2 \qquad \ldots\text{(i)}$$



Conservation of momentum gives,

$$mv_0 = mv_1 + mv_2$$

$$v_0 = v_2 + v_1 \qquad \ldots\text{(ii)}$$

From Eqs. (i) and (ii), we have

$$v_1^2 + v_2^2 + 2v_1v_2 = v_0^2$$

$$\Rightarrow \quad 2v_1v_2 = \frac{-v_0^2}{2}$$

$$\therefore (v_1 - v_2)^2 = (v_1 + v_2)^2 - 4v_1v_2 = 2v_0^2$$

or $\quad v_{rel} = \sqrt{2}v_0$

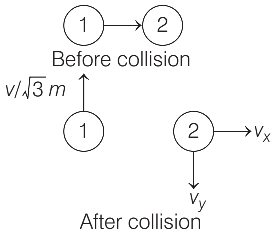
17 In x-direction,

Apply conservation of momentum, we get

$$mu_1 + 0 = mv_x$$

$$\Rightarrow \quad mv = mv_x$$

$$\Rightarrow \quad v_x = v$$



In y-direction, apply conservation of momentum, we get

$$0 + 0 = m\left(\frac{v}{\sqrt{3}}\right) - mv_y \Rightarrow v_y = \frac{v}{\sqrt{3}}$$

Velocity of second mass after collision,

$$v' = \sqrt{\left(\frac{v}{\sqrt{3}}\right)^2 + v^2} = \sqrt{\frac{4}{3}v^2}$$

or $\quad v' = \dfrac{2}{\sqrt{3}}v$

18 $U(x) = \dfrac{a}{x^{12}} - \dfrac{b}{x^6}$

$U(x = \infty) = 0$

As, $\quad F = -\dfrac{dU}{dx} = -\left[\dfrac{12a}{x^{13}} + \dfrac{6b}{x^7}\right]$

At equilibrium, $F = 0$

$\therefore \quad x^6 = \dfrac{2a}{b}$

$$\therefore U_{\text{at equilibrium}} = \frac{a}{\left(\frac{2a}{b}\right)^2} - \frac{b}{\left(\frac{2a}{b}\right)} = \frac{-b^2}{4a}$$

$$\therefore D = [U(x = \infty) - U_{\text{at equilibrium}}] = \frac{b^2}{4a}$$

19 Here, $m = 3$ kg, $t = 2$ s,

$$x = \frac{t^3}{3} \qquad \ldots(\text{i})$$

Work done,

$$W = \int dW = \int F \cdot dx \qquad \ldots(\text{ii})$$

Differentiate Eq. (i) w.r.t. time, then we get

$$\frac{dx}{dt} = \frac{3t^2}{3} \text{ or } dx = t^2dt$$

or $\quad v = t^2 \qquad \left[\text{as } \dfrac{dx}{dt} = v = \text{velocity}\right]$

or $\dfrac{dv}{dt} = 2t$ [on difference above relation]

or $\quad a = 2t \qquad \left[\text{as } \dfrac{dv}{dt} = a = \text{acceleration}\right]$

$\therefore \quad F = ma = 6t$

Hence, work done, put the value of F and dx in Eq. (ii), we get

$$W = \int_0^2 6t \times t^2dt = \left[6 \times \frac{t^4}{4}\right]_0^2 = \frac{6}{4}[2^4 - 0^4]$$

$$= \frac{3}{2} \times 16 = 24 \text{ J}$$

20 We know that change in potential energy of a system corresponding to a conservative internal force as,

$$U_f - U_i = -W = -\int \mathbf{F} \cdot d\mathbf{r}$$

Given, $\quad F = ax + bx^2$

We know that work done in stretching the rubber band by L is

$$|dW| = |Fdx|$$

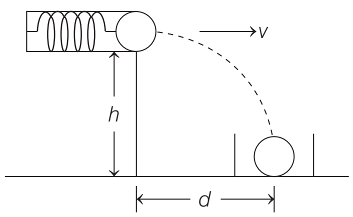
$$|W| = \int_0^L (ax + bx^2)dx$$

$$= \left[\frac{ax^2}{2}\right]_0^L + \left[\frac{bx^3}{3}\right]_0^L$$

$$= \left[\frac{aL^2}{2} - \frac{a \times (0)^2}{2}\right] + \left[\frac{b \times L^3}{3} - \frac{b \times (0)^3}{3}\right]$$

$$|W| = \frac{aL^2}{2} + \frac{bL^3}{3}$$

21 From law of conservation of energy, we have



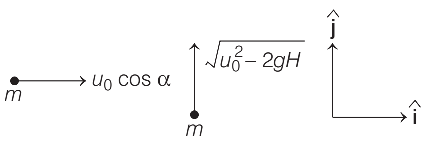
$$\frac{1}{2}mv^2 = \frac{1}{2}kx^2 \Rightarrow v = \sqrt{\frac{k}{m}}\,x$$

Time taken to fall, $t = \sqrt{\dfrac{2h}{g}}$

So, horizontal distance travelled is

$$d = vt = \sqrt{\frac{k}{m}}\,x\sqrt{\frac{2h}{g}}\,x = \sqrt{\frac{2kh}{mg}}\,x$$

22 From momentum conservation equation, we have,



$$\mathbf{p}_i = \mathbf{p}_f$$

$$\therefore m(u_0\cos\alpha)\hat{\mathbf{i}} + m(\sqrt{u_0^2 - 2gH})\hat{\mathbf{j}} = (2m)\mathbf{v} \qquad \ldots(\text{i})$$

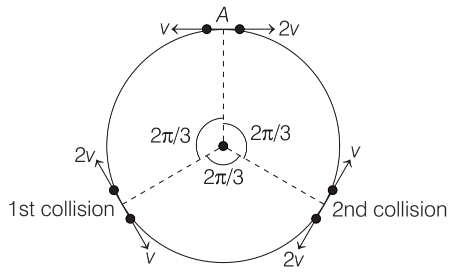
$$H = \frac{u_0^2\sin^2\alpha}{2g} \qquad \ldots(\text{ii})$$

From Eqs. (i) and (ii), we get

$$\mathbf{v} = \frac{u_0\cos\alpha}{2}\hat{\mathbf{i}} + \frac{u_0\cos\alpha}{2}\hat{\mathbf{j}}$$

Since, both components of $\mathbf{v}$ are equal. Therefore, it is making 45° with horizontal.

23 At first collision one particle having speed $2v$ will rotate 240° $\left(\text{or } \dfrac{4\pi}{3}\right)$ while other particle having speed v will rotate 120° $\left(\text{or } \dfrac{2\pi}{3}\right)$. At first collision they will exchange their velocities. Now as shown in figure, after two collisions they will again reach at point A.



24 According to the principle of conservation of momentum

$$m_1v_1 = m_2v_2 \Rightarrow \frac{v_1}{v_2} = \frac{m_2}{m_1}$$

Again, $\quad \dfrac{KE_1}{KE_2} = \dfrac{\frac{1}{2}m_1v_1^2}{\frac{1}{2}m_2v_2^2}$

$$= \frac{m_1}{m_2} \times \left(\frac{m_2}{m_1}\right)^2 = \frac{m_2}{m_1}$$

$$\therefore \quad KE \propto \frac{1}{m}$$

25 $W = \int_{\mathbf{r}_1}^{\mathbf{r}_2} \mathbf{F} \cdot d\mathbf{r}$

$$\int_{(2m,\, 3m,\, 4m)}^{(1m,\, 2m,\, 3m)} (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}}) \cdot (dx\,\hat{\mathbf{i}} + dy\,\hat{\mathbf{j}} + dz\,\hat{\mathbf{k}})$$

$$= [2x + 3y + 4z]_{(2m,\, 3m,\, 4m)}^{(1m,\, 2m,\, 3m)} = -9 \text{ J}$$

Alternate

Since, $\mathbf{F}$ = constant, we can also use

$$W = \mathbf{F} \cdot \mathbf{s}$$

Here, $\mathbf{s} = \mathbf{r}_f - \mathbf{r}_i$

$$= (\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}) - (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}})$$

$$= (-\hat{\mathbf{i}} - \hat{\mathbf{j}} - \hat{\mathbf{k}})$$

$$\therefore \quad W = (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}})(-\hat{\mathbf{i}} - \hat{\mathbf{j}} - \hat{\mathbf{k}})$$

$$= -2 - 3 - 4 = -9 \text{ J}$$